

An Extended Poincare Algebra for Linear Spinor Field Equations

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Abstract

When utilizing a cluster decomposable relativistic scattering formalism, it is most convenient that the covariant field equations take on a linear form with respect to the energy and momentum dispersion on the fields in the manner given by the Dirac form for spin $\frac{1}{2}$ systems. The general spinor formulation for arbitrary spins given in a previous paper is extended to include momentum operators. Unitary quantum mechanical representations are developed for these operators, and physical interpretations are suggested.

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1 Introduction

The Dirac equation[1] utilizes a matrix algebra to construct a linear operator relationship between the energy and the momentum. Such a linear dispersion relationship is particularly useful for constructing manifestly cluster decomposable non-perturbative scattering formalisms[2][3][4]. Expectation values of the matrices can be related to physical fluxes, but the matrices themselves commute with space-time translation generators. In a previous paper[5], a general operator representation of an extended Lorentz group which yielded the Dirac matrix representation for spin $\frac{1}{2}$ systems was developed. If the transformation is internal, this previous development is sufficient to develop quantum mechanical amplitudes that can represent general particle systems. In this paper we will extend the group to include the generators for infinitesimal space-time translations, allowing global transformations in these group parameters.

The inclusion of the expected commutation relations between energy-momenta, angular momenta, and boost operators will require non-vanishing commutation relations between energy-momenta and Γ^μ operators. In addition, group closure requires the addition of at least one additional operator to the algebra. The interpretations of this extended Poincare algebra will be explored briefly subsequent to its development.

The finite dimensional representations of the extended Poincare group will be constructed with use of the little group and complimentary group of transformations on the standard state vectors in the construction of particle-like unitary representations for spin states. In addition, a single-particle wave equation can be developed for configuration space eigenstates of the operator $\hat{\Gamma}^\mu \hat{P}_\mu$,

$$-\Gamma^\mu i\partial_\mu \psi(x) = \lambda \psi(x) \quad (1.1)$$

which implies that

$$\partial_\mu \left[\overline{\psi(x)} \Gamma^\mu \psi(x) \right] = 0. \quad (1.2)$$

The conserved current defined in Equation 1.2 need not be the probability current, since the spinor metric is generally not directly related to the Γ matrices. However, scattering equations can be developed to express the evolution of the physical parameter represented by this operator.

Finally, we will see that transformations by the new extended group parameters mixes what would be the mass parameter for traditional particle states with the new transverse mass parameter generated by the additional operator. We will briefly examine the behavior of states transformed by these extended group operators.

2 Extended Lorentz Group

As explained in the introduction, we consider it advantageous to extend the previously developed extended Lorentz group to include the usual Poincare algebra, but generally require that the form $\hat{\Gamma}^\mu \hat{P}_\mu$

is a scalar operator. In a previous paper [5], spinor and matrix representations were developed for the generators conjugate to the 10 group elements given by 3 rotations θ_k , 3 Lorentz boosts v_k , and the 4 group parameters associated with the generators $\hat{\Gamma}^\mu$ given by $\vec{\omega}$. For completeness, we will here briefly review those relationships.

2.1 Extended Lorentz Group Commutation Relations

The commutation relationships between the generators of this extended Lorentz group are given by

$$[J_j, J_k] = i \epsilon_{jkm} J_m \quad (2.1)$$

$$[J_j, K_k] = i \epsilon_{jkm} K_m \quad (2.2)$$

$$[K_j, K_k] = -i \epsilon_{jkm} J_m \quad (2.3)$$

$$[\Gamma^0, \Gamma^k] = i K_k \quad (2.4)$$

$$[\Gamma^0, J_k] = 0 \quad (2.5)$$

$$[\Gamma^0, K_k] = -i \Gamma^k \quad (2.6)$$

$$[\Gamma^j, \Gamma^k] = -i \epsilon_{jkm} J_m \quad (2.7)$$

$$[\Gamma^j, J_k] = i \epsilon_{jkm} \Gamma^m \quad (2.8)$$

$$[\Gamma^j, K_k] = -i \delta_{jk} \Gamma^0 \quad (2.9)$$

An extended Lorentz group Casimir operator can be constructed in the form

$$C_\Lambda = \underline{J} \cdot \underline{J} - \underline{K} \cdot \underline{K} + \Gamma^0 \Gamma^0 - \underline{\Gamma} \cdot \underline{\Gamma}. \quad (2.10)$$

2.2 Spinor Equations

We found that the most elegant way to construct general finite dimensional representations was by utilizing the formalism of spinors. Operations in the form of raising and lowering operators are most conveniently expressed in terms of spinors. For convenience, define $\Delta_k^{(\pm)}$ as follows:

$$\Delta_k^{(\pm)} \equiv \Gamma^k (\pm) i K_k. \quad (2.11)$$

We chose to construct eigenstates of C_Λ , Γ^0 , J^2 , and J_z . To develop a basis of states, it was convenient to construct an operator which raises and lowers eigenvalues of the operator Γ^0 . This operator is given by

$$\Delta_J^{(\pm)} \equiv \underline{J} \cdot \underline{\Delta}^{(\pm)} \quad (2.12)$$

These definitions allow us to express general states and operators in terms of the spinors:

$$\begin{aligned} \psi_{\gamma, M}^{\Lambda, J} &= A^{\Lambda J} \sqrt{\frac{(J-M)!}{(J+M)!(2J)!}} [x-y]^{\Lambda-J} \chi_+^{(+M+\gamma)} \chi_+^{(-M-\gamma)} \times \\ &\quad \left[\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right]^{J+M} x^{J-\gamma} y^{J+\gamma} \Bigg|_{\substack{x = \chi_+^{(+)} \chi_-^{(-)} \\ y = \chi_-^{(+)} \chi_+^{(-)}}} \end{aligned} \quad (2.13)$$

$$\hat{J}^2 \psi_{\gamma,M}^{\Lambda,J} = J(J+1) \psi_{\gamma,M}^{\Lambda,J} \quad , \quad \hat{C} \psi_{\gamma,M}^{\Lambda,J} = 2\Lambda(\Lambda+2) \psi_{\gamma,M}^{\Lambda,J} \quad (2.14)$$

$$\hat{J}_z \psi_{\gamma,M}^{\Lambda,J} = M \psi_{\gamma,M}^{\Lambda,J} \quad , \quad \hat{\Gamma}^0 \psi_{\gamma,M}^{\Lambda,J} = \gamma \psi_{\gamma,M}^{\Lambda,J} \quad (2.15)$$

$$\hat{J}_{\pm} \psi_{\gamma,M}^{\Lambda,J} = \sqrt{(J \pm M + 1)(J \mp M)} \psi_{\gamma,M \pm 1}^{\Lambda,J} \quad , \quad \hat{\Delta}^{(\pm)} \psi_{\gamma,M}^{\Lambda,J} = (\pm)(\Lambda+1)[J(\mp)\gamma] \psi_{\gamma \pm 1,M}^{\Lambda,J} \quad (2.16)$$

2.3 Spinor metric

Invariant amplitudes that can be interpreted as probability amplitudes are defined using dual spinors, so that under transformations the inner product is a scalar

$$\begin{aligned} < \bar{\psi} | \phi > = < \bar{\psi}' | \phi' > \\ \psi_a^\dagger g_{ab} \phi_b &= (D_{ca} \psi_a)^\dagger g_{cd} (D_{db} \psi_b) \end{aligned} \quad (2.17)$$

The metric satisfies

$$g_{\gamma}^{\Lambda J} = (-)^{\Lambda - \gamma}, \quad (2.18)$$

which will be associated with $\bar{\psi}$ in matrix element evaluations.

2.4 Matrix Representation for $\Lambda = 1$

As a specific example, the matrix representation corresponding to $\Lambda = 1$ will be explicitly demonstrated below:

$$\begin{aligned} \mathbf{\Gamma}^0 &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} & \mathbf{\Gamma}^k &= \frac{1}{2} \begin{pmatrix} 0 & 2v_k^T & 0 & 2v_k^T \\ -v_k & 0 & \mathbf{J}_k & 0 \\ 0 & -2\mathbf{J}_k & 0 & 2\mathbf{J}_k \\ -v_k & 0 & -\mathbf{J}_k & 0 \end{pmatrix} \\ \mathbf{J}_k &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{J}_k & 0 & 0 \\ 0 & 0 & \mathbf{J}_k & 0 \\ 0 & 0 & 0 & \mathbf{J}_k \end{pmatrix} & \mathbf{K}_k &= \frac{1}{2} \begin{pmatrix} 0 & -2v_k^T & 0 & 2v_k^T \\ -v_k & 0 & \mathbf{J}_k & 0 \\ 0 & 2\mathbf{J}_k & 0 & 2\mathbf{J}_k \\ v_k & 0 & \mathbf{J}_k & 0 \end{pmatrix} \\ \mathbf{g} &= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{aligned} \quad (2.19)$$

where

$$\begin{aligned} v_z &\equiv \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} & v_x &\equiv \frac{\sqrt{2}}{2} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} & v_y &\equiv -\frac{\sqrt{2}}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \\ \mathbf{J}_z &\equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} & \mathbf{J}_x &\equiv \frac{\sqrt{2}}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} & \mathbf{J}_y &\equiv \frac{\sqrt{2}}{2} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \end{aligned} \quad (2.20)$$

The representation generated by these matrices is seen to be finite dimensional, but not unitary (analogous to the Dirac representation for spin $\frac{1}{2}$ systems).

3 An Extended Poincare Group

3.1 Extended Poincare Group Closure

The equations presented thus far are valid for internal transformations on systems. We will next attempt to minimally expand the group structure to include space-time translations, in order to develop a group that can be used for global transformations. An attempt to only include the momentum operators does not produced a closed group structure, due to Jacobi relations of the sort given by $[P_j, [\Gamma^0, \Gamma^k]]$. The non-vanishing of this commutator in the Jacobi identity implies a non-vanishing commutator between Γ^μ and P_ν , and that this commutator connect to an operator which then commutes with Γ^μ to yield a P_ν . Since the momentum operators self-commute, this requires the introduction of at least one additional operator that we will refer to as \mathcal{G} .

3.2 A Closed Set of Extended Poincare Operators

The non-vanishing commutators involving the operators \hat{P}_μ and $\hat{\mathcal{G}}$ that satisfy the Jacobi identities are given by

$$[J_j, P_k] = i \epsilon_{jkm} P_m \quad (3.1)$$

$$[K_j, P_0] = -i P_j \quad (3.2)$$

$$[K_j, P_k] = -i \delta_{jk} P_0 \quad (3.3)$$

$$[\Gamma^\mu, P_\nu] = i \delta_\nu^\mu \mathcal{G} \quad (3.4)$$

$$[\Gamma^\mu, \mathcal{G}] = i \eta^{\mu\nu} P_\nu \quad (3.5)$$

We can construct an extended Poincare group Casimir operator given by

$$\mathcal{C}_\mu \equiv \mathcal{G}^2 - \eta^{\beta\nu} P_\beta P_\nu. \quad (3.6)$$

The extended Lorentz subgroup Casimir operator defined in Equation 2.10 has the following nontrivial commutation relations:

$$[\mathcal{G}, C_\Lambda] = i (\Gamma^\mu P_\mu + P_\mu \Gamma^\mu) \quad (3.7)$$

$$[P_0, C_\Lambda] = -i \left(\Gamma^0 \mathcal{G} + \mathcal{G} \Gamma^0 - \sum_j K_j P_j + P_j K_j \right) \quad (3.8)$$

$$[P_j, C_\Lambda] = i (\Gamma^k \mathcal{G} + \mathcal{G} \Gamma^k - K_j P_0 - P_0 K_j + \epsilon_{jkm} (J_k P_m + P_m J_k)) \quad (3.9)$$

3.3 Extended Poincare Group metrics

In general, a group metric can be developed from the adjoint representation in terms of the structure constants. The non-vanishing group metric elements generated by the structure constants of this extended

Poincare group are given by

$$\eta_{J_m J_n}^{(P)} = -8 \delta_{m,n} \quad \eta_{K_m K_n}^{(P)} = +8 \delta_{m,n} \quad (3.10)$$

$$\eta_{\Gamma^\mu \Gamma^\nu}^{(P)} = 8 \eta^{\mu\nu} \quad (3.11)$$

where $\eta^{\mu\nu}$ is the usual Minkowski metric of the Lorentz group. This metric is seen to be non-trivially generated by the extended Γ algebra.

3.4 Local Factors

For a general ray representation of a quantum mechanical system, group transformations on a quantum state vector can introduce additional phase factors;

$$U(\underline{b}) U(\underline{a}) = e^{i\zeta(\underline{b};\underline{a})} U(\underline{\phi}(\underline{b};\underline{a})) \quad (3.12)$$

where ζ is the local exponent and ϕ represents the general group multiplication element. The behavior of the local exponent under group transformations can in general introduce local factors which are c-numbers into the algebra of the generators. These local factors can have physical significance, as in the case of the Galilean group for non-relativistic transformations[6]. We will briefly explore the local factors for this extended Poincare group.

All local factors for this group can be shown to vanish except those in the following list of commutators: Equations 2.2 and 2.4, where the term on the right hand sides is shifted by $K_m - \xi_{\Gamma^m, \Gamma^0}$; Equations 2.8 and 2.6, where the term on the right hand sides is shifted by $\Gamma^m + \xi_{K_m, \Gamma^0}$; Equation 2.9, where the term on the right hand side is shifted by $\Gamma^0 + \xi_{K_z, \Gamma^z}$; Equations 3.1, 3.2, and 3.5, where the term on the right hand sides is shifted by $P_m + \xi_{\Gamma^m, \mathcal{G}}$; Equations 3.3 and 3.5, where the term on the right hand sides is shifted by $-P_0 + \xi_{\Gamma^0, \mathcal{G}}$; Equation 3.4, where the term on the right hand side is shifted by $\mathcal{G} + \xi_{\Gamma^0, P_0}$. The local factors ξ appear in precisely the appropriate manner such that they can be absorbed into re-definitions of K_m, Γ^μ, P_μ , and \mathcal{G} , eliminating their appearance in the commutation relations. Henceforth, we will work with a representation in which all local factors have been eliminated.

3.5 Unitary Representations of the Extended Poincare Group

A natural set of discrete basis states for the extended Poincare algebra is specified in terms of the group Casimir, the extended Lorentz subgroup Casimir, and mutually commuting eigenvalues of Γ^0 , J^2 , and J_z given by $|\mu^2, \Lambda, \gamma, J, M\rangle$. Here μ^2 represents the eigenvalue of the extended Poincare group Casimir operator, and the eigenvalue of the extended Lorentz subgroup Casimir can be expressed in terms of Λ . If we are to construct states that have correspondence with the usual particle states, we need to examine

eigenstates of \hat{P}_0 . We have several choices on how to choose mutually commuting parameters, for instance $|\mu^2, p_{(s)0}, \gamma, J, M\rangle$ or $|p_{(s)0}, \mathcal{Q}_{(s)}, \gamma, J, M\rangle$. We will choose states of the form

$$|\sqrt{-\vec{p}_{(s)} \cdot \vec{p}_{(s)}}, \mathcal{Q}_{(s)} = 0, \gamma, J, M\rangle$$

which have vanishing value when operated on by \mathcal{G} as the standard particle states from which we can build unitary representations. States of finite momenta are constructed using pure Lorentz boosts upon the standard state vector:

$$|\underline{u}, \mu, \gamma, J, M\rangle \equiv L(\underline{u}) \left| \sqrt{-\vec{p}_{(s)} \cdot \vec{p}_{(s)}}, \mathcal{Q}_{(s)} = 0, \gamma, J, M \right\rangle \quad (3.13)$$

where $\mu^2 = -\vec{p}_{(s)} \cdot \vec{p}_{(s)}$ and $\vec{u} \cdot \vec{u} = -1$ for massive states. Unitary transformations involving general Lorentz transformations $\mathbf{\Lambda}$ give

$$U(\mathbf{\Lambda})|\underline{u}, \mu, \gamma, J, M\rangle = \sum_{M'=-J}^J |\underline{\mathbf{\Lambda}}\underline{u}, \mu, \gamma, J, M'\rangle D_{M',M}^{(J)}(R_W(\mathbf{\Lambda}, \underline{u})) \quad (3.14)$$

where $R_W(\mathbf{\Lambda}, \underline{u}) = L^{-1}(\underline{\mathbf{\Lambda}}\underline{u})\mathbf{\Lambda}L(\underline{u})$ is the usual Wigner rotation[7]. This represents the little group element for the Lorentz subgroup for the standard state vector $\vec{p}_{(s)}$.

For more general transformations by group elements \underline{g} conjugate to the standard state group element $\underline{g}_{(s)}$, the little group of transformations will leave the standard state group element invariant, $\mathcal{R}_{\underline{g}_{(s)}} = \underline{g}_{(s)}$. Since angular momenta and Γ^0 are hermitian operators, operations involving this subgroup of transformations will be generalized rotations on the standard state vector that will leave the standard state group element invariant. Using the complimentary group of transformations, boosted states can be defined:

$$|\underline{g}, \mu^2, \Lambda, \gamma, J, M\rangle \equiv U(C(\underline{g}))|\underline{g}_{(s)}, \mu^2, \Lambda, \gamma, J, M\rangle \quad (3.15)$$

where $C(\underline{g})\underline{g}_{(s)} = \underline{g}$. A unitary representation can then be constructed using

$$U(\mathcal{M})|\underline{g}, \mu^2, \Lambda, \gamma, J, M\rangle = \sum_{\gamma'=-J}^J \sum_{M'=-J}^J |\mathcal{M}\underline{g}, \mu^2, \Lambda, \gamma', J, M'\rangle D_{\gamma'M';\gamma M}^{(\Lambda J)}(\mathcal{R}(\mathcal{M}, \underline{g})), \quad (3.16)$$

where the little group element satisfies

$$\mathcal{R}(\mathcal{M}, \underline{g}) \equiv C^{-1}(\mathcal{M}\underline{g})\mathcal{M}C(\underline{g}). \quad (3.17)$$

The group structure of this extended Lorentz and Poincare group will be explored in subsequent papers.

3.6 Linear Wave Equation for Single Particle States

A primary motivation for this work was to develop general finite dimensional expressions of the operator $\Gamma^\mu P_\mu$. Eigenstates of this operator should give linear operator dispersion for energy and momenta in a

wave equation. It is straightforward to calculate the commutators of the various group generators with this operator:

$$[J_k, \Gamma^\mu P_\mu] = 0 \quad (3.18)$$

$$[K_k, \Gamma^\mu P_\mu] = 0 \quad (3.19)$$

$$[\Gamma^k, \Gamma^\mu P_\mu] = i\Gamma^k \mathcal{G} + i\epsilon_{kmn} J_m P_n - iK_k P_0 \quad (3.20)$$

$$[\Gamma^0, \Gamma^\mu P_\mu] = i\Gamma^0 \mathcal{G} + i \sum_j K_j P_j \quad (3.21)$$

$$[P_\beta, \Gamma^\mu P_\mu] = -i\mathcal{G} P_\beta \quad (3.22)$$

$$[\mathcal{G}, \Gamma^\mu P_\mu] = -i\eta^{\beta\nu} P_\beta P_\nu \quad (3.23)$$

Since pure Lorentz transformations commute with the operator $\Gamma^\mu P_\mu$ from Equations 3.18 and 3.19, we can define a mixed wavefunction using Equation 3.13

$$\langle \mu^2, \Lambda, \gamma, J, M | \Gamma^\mu P_\mu | \underline{\mu}, \mu, \gamma, J, M \rangle = \gamma \mu \langle \mu^2, \Lambda, \gamma, J, M | \underline{\mu}, \mu, \gamma, J, M \rangle \quad (3.24)$$

to obtain a linear wave equation given by

$$\mathbf{\Gamma}^\mu \hat{P}_\mu \psi_{\gamma M}^\Lambda(\mu \underline{\mu}) = \gamma \mu \psi_{\gamma M}^\Lambda(\mu \underline{\mu}) \quad (3.25)$$

where

$$\psi_{\gamma M}^\Lambda(\mu \underline{\mu}) \equiv \langle \mu^2, \Lambda, \gamma, J, M | \underline{\mu}, \mu, \gamma, J, M \rangle. \quad (3.26)$$

In a configuration space basis, this can be written as

$$\mathbf{\Gamma}^\mu (-i\partial_\mu) \psi_{\gamma M}^\Lambda(x) = \gamma \mu \psi_{\gamma M}^\Lambda(x) \quad (3.27)$$

to give the eigenvalue differential equation.

It is clear that since for two non-interacting subsystems the components of the energy-momentum of clusters are additive, such linear dispersions make explicit clustering properties more apparent. It should then be straightforward to include the kinematic variables of a non-interacting cluster in a purely parametric way when calculating the dynamics of an off-shell, off-diagonal subsystem.

3.7 Finite Extended Parameter Transforms

We end by examining finite transformations in the extended group parameters. To examine transformations on the generators that define the minimal Poincare extension of the extended Lorentz group, we will express the group parameter conjugate to the operators $\hat{\Gamma}^\mu$ in terms of a magnitude and direction, $\omega_\mu \equiv \omega u_\mu$. One can directly demonstrate that the commutation relations Equations 3.4 and 3.5 imply that for time-like ($\vec{u} \cdot \vec{u} = -1$) or space-like ($\vec{u} \cdot \vec{u} = +1$) transformations

$$e^{i\omega_\mu \hat{\Gamma}^\mu} \hat{\mathcal{G}} e^{-i\omega_\mu \hat{\Gamma}^\mu} = \cos(\sqrt{-\vec{\omega} \cdot \vec{\omega}}) \hat{\mathcal{G}} + \frac{\sin(\sqrt{-\vec{\omega} \cdot \vec{\omega}})}{\sqrt{-\vec{\omega} \cdot \vec{\omega}}} \omega_\mu \eta^{\mu\nu} \hat{P}_\nu \quad (3.28)$$

$$e^{i\omega_\mu \hat{\Gamma}^\mu} \hat{P}_\beta e^{-i\omega_\mu \hat{\Gamma}^\mu} = \omega_\beta \frac{\sin(\sqrt{-\vec{\omega} \cdot \vec{\omega}})}{(\sqrt{-\vec{\omega} \cdot \vec{\omega}})} \hat{\mathcal{G}} + \left[\delta_\beta^\nu + \frac{\omega_\beta \omega^\nu}{\vec{\omega} \cdot \vec{\omega}} (\cos(\sqrt{-\vec{\omega} \cdot \vec{\omega}}) - 1) \right] \hat{P}_\nu. \quad (3.29)$$

We can see that for eigenstates of $\hat{\Gamma}^0$ such as the finite dimensional representations discussed previously, the time-like transformations mix eigenvalues of \hat{P}_0 (masses) with eigenvalues of $\hat{\mathcal{G}}$. The mass values oscillate under variations in the parameter ω . If the group elements ω are discrete, then the discrete set of masses will mix into each other and eigenvalues of \mathcal{G} under operations using finite transformations in group elements conjugate to Γ^0 . In this sense, the operator \mathcal{G} which is necessary for group closure can be interpreted as a transverse mass operator.

Similarly, one can examine the non-trivial transformations by the operators \mathcal{G} and \hat{P}_0 . The operators $\hat{\Gamma}^\mu$ transform as follows:

$$e^{i\alpha \hat{\mathcal{G}}} \hat{\Gamma}^\mu e^{-i\alpha \hat{\mathcal{G}}} = \hat{\Gamma}^\mu + \alpha \eta^{\mu\nu} \hat{P}_\nu \quad (3.30)$$

$$e^{ia^\beta \hat{P}_\beta} \hat{\Gamma}^\mu e^{-ia^\beta \hat{P}_\beta} = \hat{\Gamma}^\mu + a^\mu \hat{\mathcal{G}} \quad (3.31)$$

These equations directly indicate that a discrete finite dimensional representation that can be used to model the behaviors of the usual particles in quantum field theory cannot be constructed if $\hat{\mathcal{G}}$ has non-vanishing expectation values, or if there is a finite transformation $\alpha \neq 0$ upon such a state (unless the group parameters α are discrete).

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References

- [1] P.A.M. Dirac, Proc. Roy. Soc. (London), A117, 610 (1928); *ibid*, A118, 351 (1928)
- [2] J.V. Lindesay, A.J. Markevich, H.P. Noyes, and G. Pastrana, Phys. Rev. D33, 2339 (1986)
- [3] M. Alfred, P. Kwizera, J.V. Lindesay, and H.P. Noyes, A Non-Perturbative, Finite Particle Number Approach to Relativistic Scattering Theory, SLAC-PUB-8821, hep-th/0105241 (2001)
- [4] J. Lindesay and H.P. Noyes, Non-Perturbative, Unitary Quantum-Particle Scattering Amplitudes, SLAC-PUB-9164, hep-th/0203262 (2002)
- [5] J. Lindesay, Linear Spinor Field Equations for Arbitrary Spins, math-ph/0308003 (2003)

- [6] H.L. Morrison and J.V. Lindesay, Galilean Presymmetry and the Quantization of Circulation, Journal of Low Temperature Physics 26, 899-907 (1977)
- [7] E.P. Wigner, Ann. Math. 40, 149 (1939)